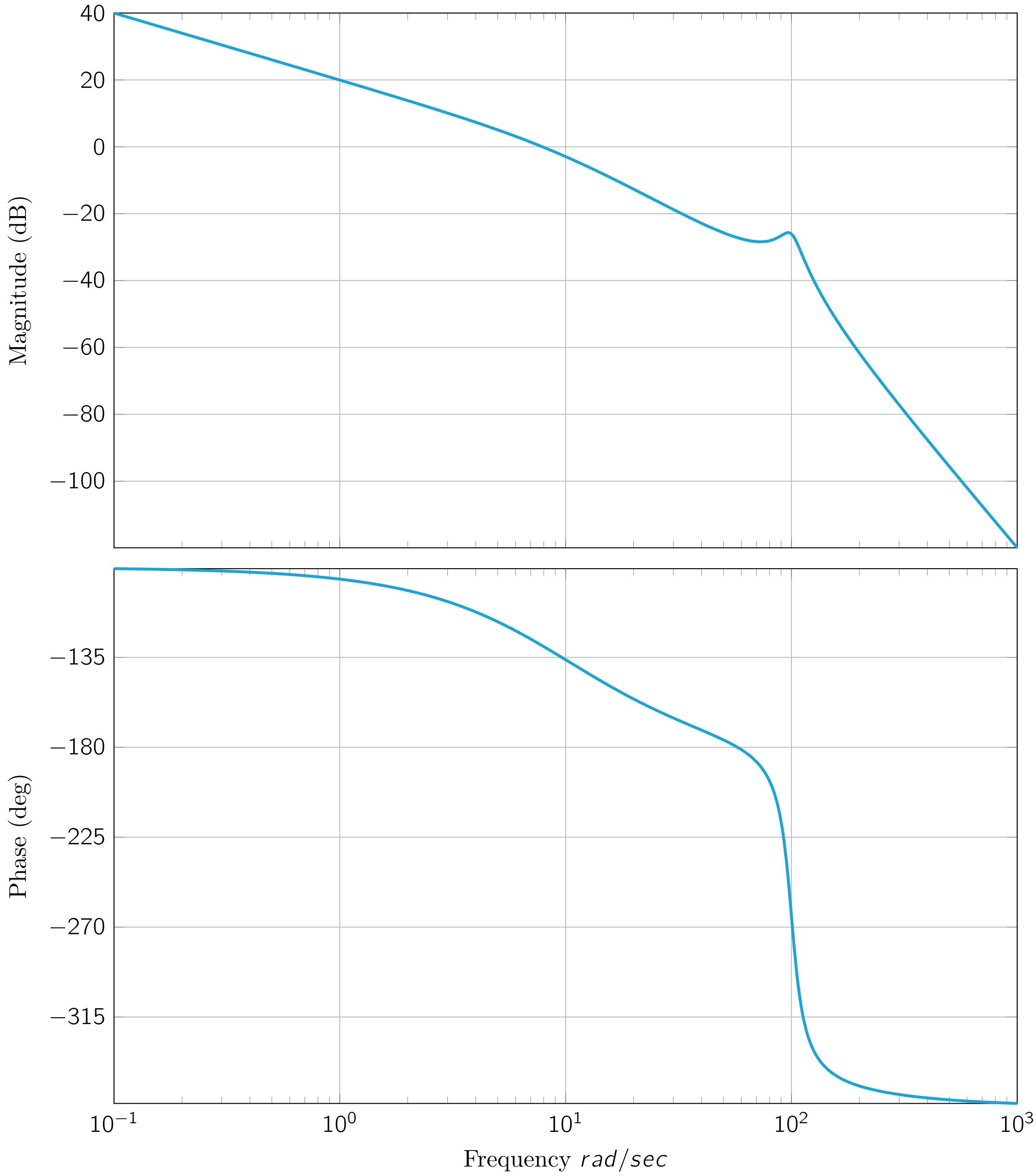
Control Systems : Set 5 : Loopshaping (1) - Solutions

Prob 1 | Consider the Bode plot given below. Estimate a model for the system.



We see that as the frequency goes to zero, the slope of the system is $-20dB/dec$ and the phase is $-90°$. This tells us that there is one integrator.
In Bode form, our transfer function will be

$$G(j\omega) = K_0 \frac{1}{j\omega} \prod \left(\frac{j\omega}{p_i} + 1 \right)^{\pm 1}$$

and so at the low frequency of $\omega = 10^{-1}$, where we see that the gain is $40dB$ we have the relationship

$$\begin{aligned} 20\log_{10}|G(j\omega)| = 20\log_{10} K_0 - 20\log_{10}\omega = 20\log_{10} K_0 - 20\log_{10} 0.1 = 40 \\ \rightarrow\ 20\log_{10} K_0 = 40 + 20\log_{10} 0.1 = 20 \\ \rightarrow\ K_0 = 10 \end{aligned}$$

The phase drops by $90°$, with the halfway point being at $10r/s$. In addition, we can see that the slope changes from $-2dB/dec$ to $-40dB/dec$ at $10r/s$, which tells us that there is a pole at $s = -10$.
The $180°$ drop in phase accompanied by the $-40dB/dec$ drop in the slope of the magnitude plot tells us that there is a second order pole at a frequency of $\omega_n = 100r/s$. To estimate the damping ratio, we measure that the size of the resonance is approximately $10dB$ (difference between the magnitude plot, and a straight-line approximation). Using the relationship below, we can estimate the damping ratio

$$\begin{aligned} 20\log_{10}\frac{1}{2\zeta} \approx 10 \\ \rightarrow\ \zeta \approx 0.16 \end{aligned}$$

Putting this together, we get the model

$$\begin{aligned} G(s) &\approx 10\frac{1}{(s/10+1)\left(\frac{s}{100}^2 + 2\zeta\frac{s}{100} + 1\right)s} \\ &= \frac{1,000,000}{(s+10)\left(s^2 + 200\zeta s + 10,000\right)s} \end{aligned}$$

Matlab | There are several ways to enter transfer functions in Matlab - tf, zpk and ss. Use the help command in Matlab to see how they are used.
In addition, it is possible to enter transfer functions directly as a function of the Laplace variable

```
s = tf('s')
G = (s-3)/(s^2 + 3*s + 34)
```

You can then manipulate transfer functions algebraically, e.g., G1 * G2 + G3.

Computing standard closed-loop transfer functions can be done with the command feedback.

Some commands that you'll find useful

bode	Create Bode plot
nyquist	Create Nyquist plot
step	Compute unit step function.
	To compute the ramp response, just integrate the step input by multiplying by $1/s$
	$step(G)$ Step response
	$step(G/s)$ Ramp response
impulse	Compute unit impulse function
lsim	Compute response to arbitrary input signal

Prob 2 | Sketch the bode plot for the system

$$G(s) = \frac{10}{s\,(s+10)}$$

Check your answer using the Matlab command bode

a) Put into Bode form
Order poles/zeros from lowest to largest frequency:

$$G(jw) = 1 \cdot (jw)^{-1} \cdot \left(\frac{jw}{10} + 1\right)^{-1}$$

b) Compute value at low-frequencies
There is one integrator. The bode plot will go to infinity asymptotically with a slope of -20 dB/dec as $\omega \to 0$
Compute a point on the asymptote for some value of ω. We choose $\omega = 0.1$ because this is two orders of magnitude smaller than the smallest pole or zero.

$$20\log_{10} G(j0.1) = 20\log_{10} 1 + (-1)20\log_{10}(j0.1) = 20dB$$

c) Compute the phase at low frequencies

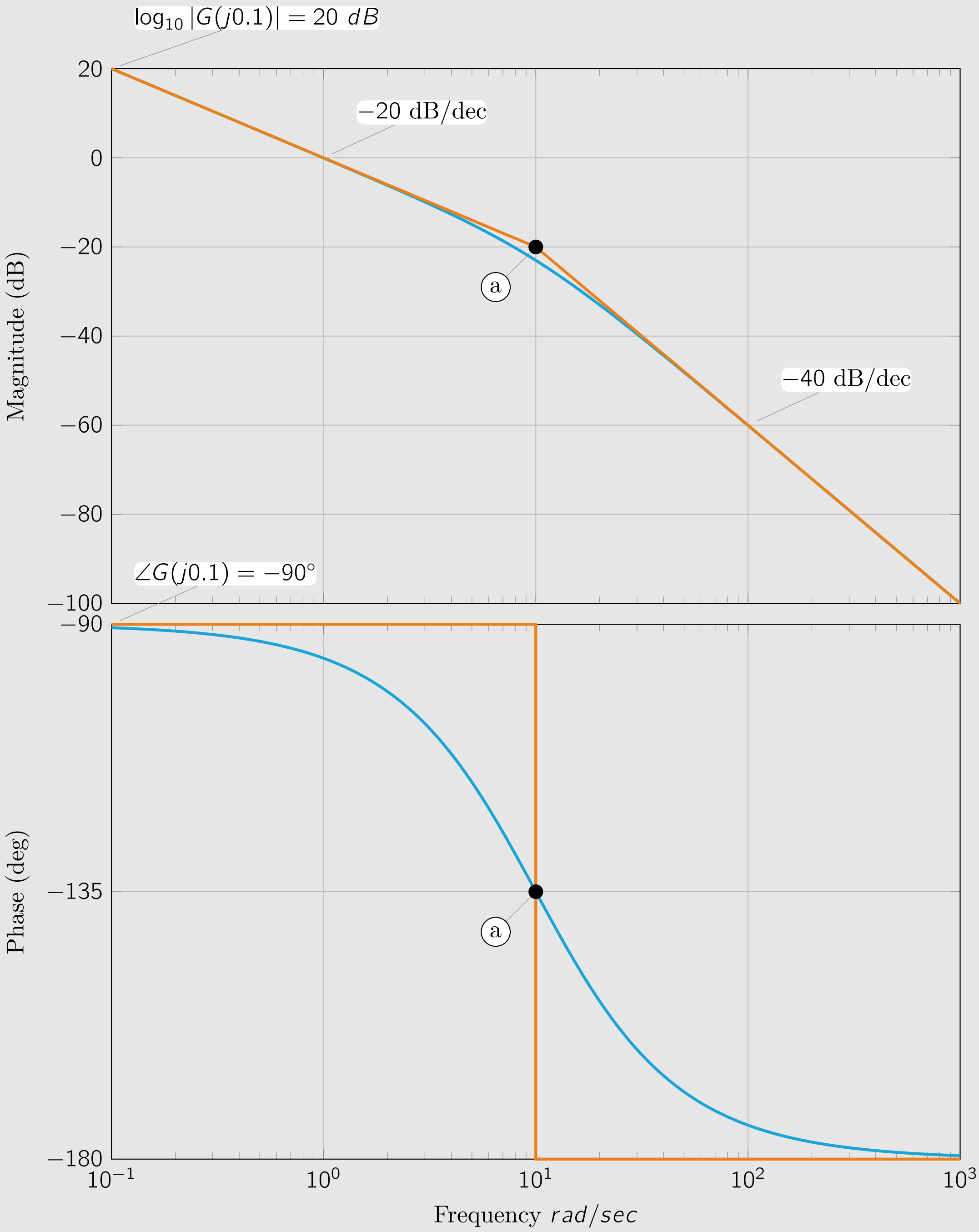
$$\begin{aligned}
\lim_{\omega\to 0} \angle G(j\omega) =& 0 && 0^\circ \text{ if } K_0 > 0 \text{ else } 180^\circ \\
& -90 \cdot 1 && -90 \cdot (\text{Num integrators}) \\
& +90 \cdot 0 && 90 \cdot (\text{Num zeros at zero}) \\
& -180 \cdot 0 && -180 \cdot (\text{Num of RHP poles}) \\
& +180 \cdot 0 && 180 \cdot (\text{Num of RHP zeros}) \\
=& -90 &&
\end{aligned}$$

d) Compute impact of each pole and zero
Compute impact of each pole / zero on magnitude and phase from low to high frequency.

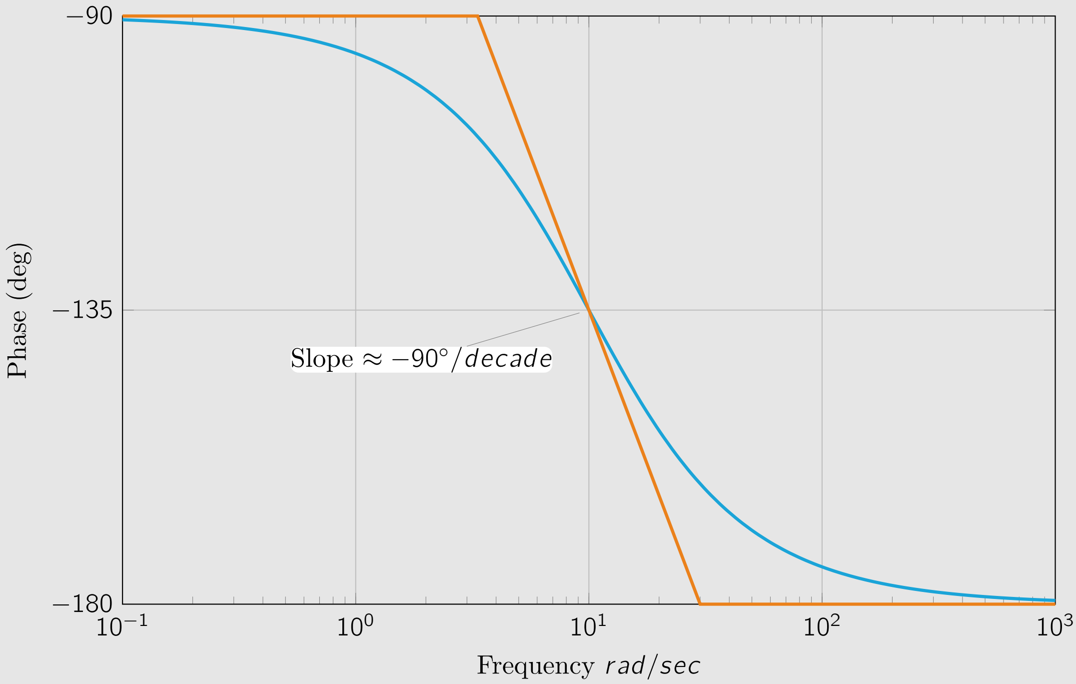
		Freq	Pole/Zero	Order	ΔdMag	ΔPhase
a	$\left(\frac{jw}{10}+1\right)^{-1}$	10	LHP pole	1^{st}	-20	-90

e) Sketch straight-line approximation

Straight-line approximation (orange). True Bode plot (blue)



f) Improve accuracy of phase plot if required
Straight-line approximation (orange). True Bode plot (blue)



Prob 3 | Sketch the bode plot for the system

$$G(s) = \frac{5}{s\,(s+2)\,(s+1)}$$

Check your answer using the Matlab command bode

a) Put into Bode form
Order poles/zeros from lowest to largest frequency:

$$G(jw) = 2.5 \cdot (jw)^{-1} \cdot \left(\frac{jw}{1} + 1\right)^{-1} \cdot \left(\frac{jw}{2} + 1\right)^{-1}$$

b) Compute value at low-frequencies
There is one integrator. The bode plot will go to infinity asymptotically with a slope of -20 dB/dec as $\omega \to 0$
Compute a point on the asymptote for some value of ω. We choose $\omega = 0.01$ because this is two orders of magnitude smaller than the smallest pole or zero.

$$20\log_{10} G(j0.01) = 20\log_{10} 2.5 + (-1)20\log_{10}(j0.01) = 47.96dB$$

c) Compute the phase at low frequencies

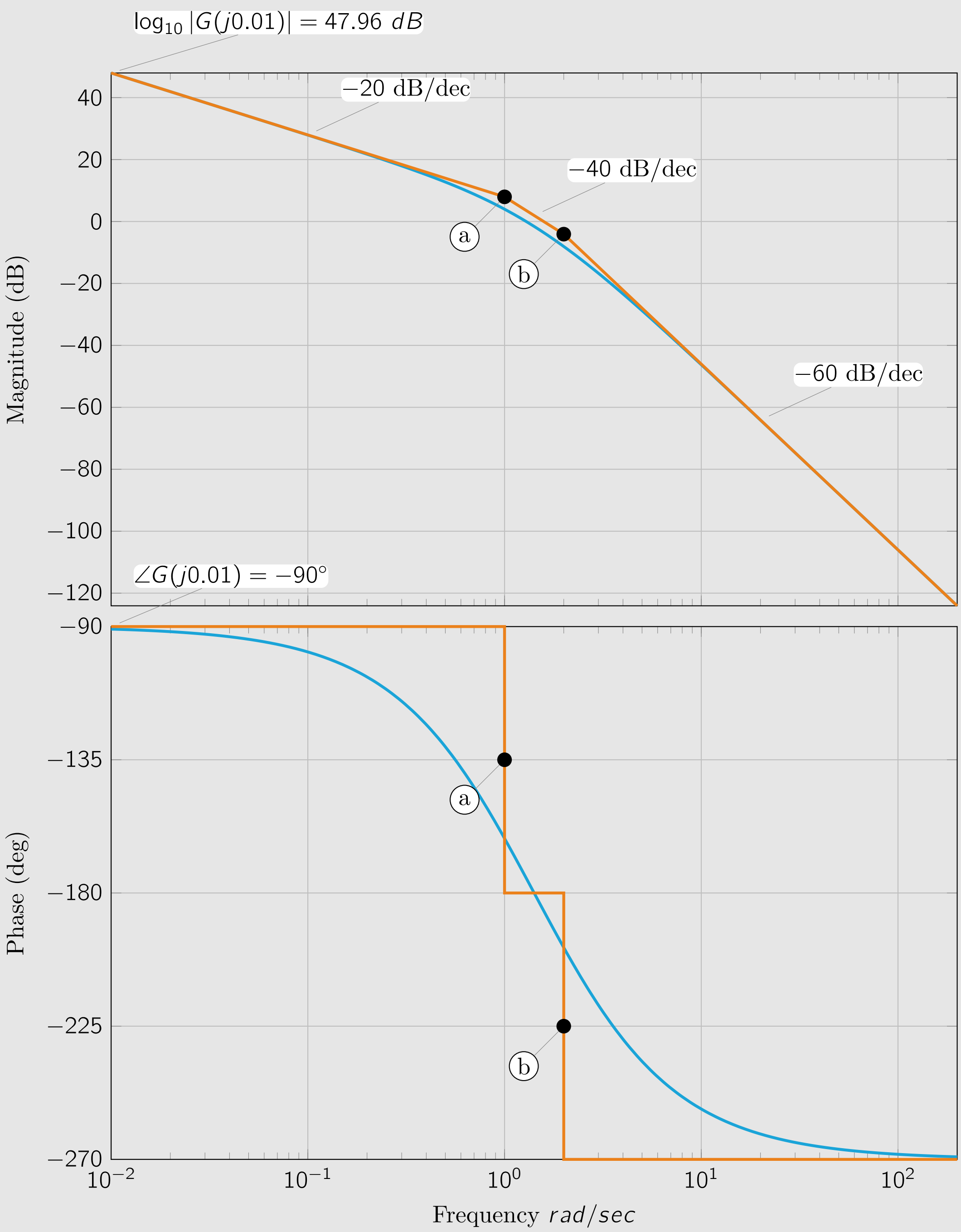
$$\begin{aligned}
\lim_{\omega \to 0} \angle G(j\omega) =& 0 && 0^\circ \text{ if } K_0 > 0 \text{ else } 180^\circ \\
& -90 \cdot 1 && -90 \cdot \text{(Num integrators)} \\
& +90 \cdot 0 && 90 \cdot \text{(Num zeros at zero)} \\
& -180 \cdot 0 && -180 \cdot \text{(Num of RHP poles)} \\
& +180 \cdot 0 && 180 \cdot \text{(Num of RHP zeros)} \\
=& -90
\end{aligned}$$

d) Compute impact of each pole and zero
Compute impact of each pole / zero on magnitude and phase from low to high frequency.

		Freq	Pole/Zero	Order	ΔdMag	ΔPhase
a	$\left(\frac{jw}{1} + 1\right)^{-1}$	1	LHP pole	1^{st}	-20	-90
b	$\left(\frac{jw}{2} + 1\right)^{-1}$	2	LHP pole	1^{st}	-20	-90

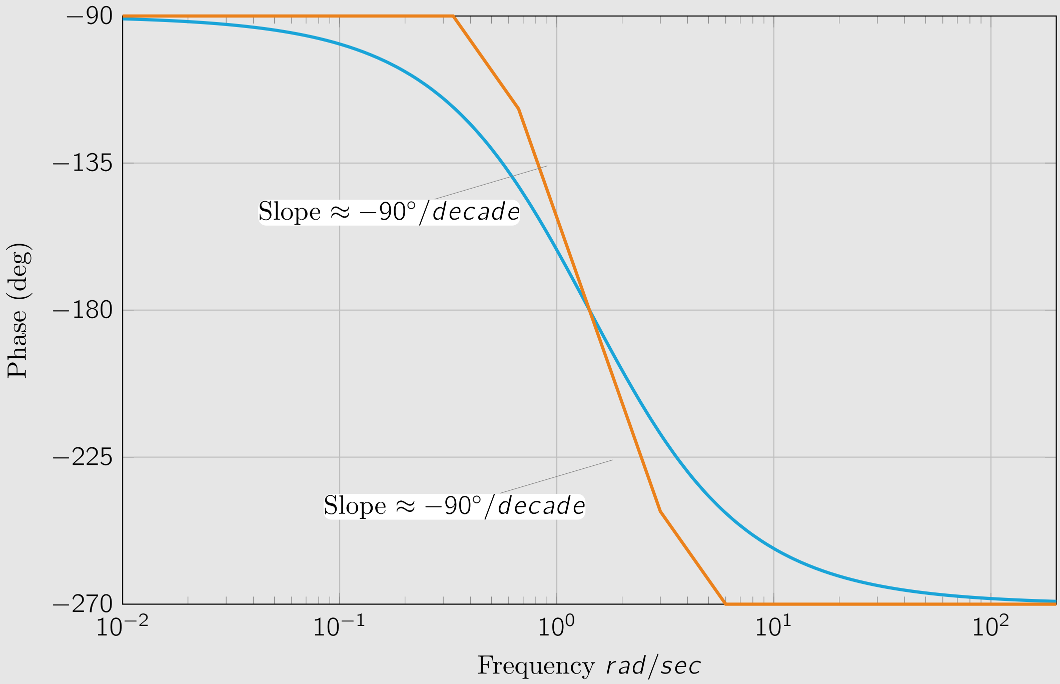
e) Sketch straight-line approximation

Straight-line approximation (orange). True Bode plot (blue)



f) Improve accuracy of phase plot if required

Straight-line approximation (orange). True Bode plot (blue)



Prob 4 | Sketch the bode plot for the system

$$G(s) = \frac{s}{s^2 + 2s + 5}$$

Check your answer using the Matlab command bode

a) Put into Bode form

Order poles/zeros from lowest to largest frequency:

$$G(jw) = 0.2 \cdot (jw)^1 \cdot \left[\left(\frac{jw}{2.236} \right)^2 + 2 \cdot (0.4636) \frac{jw}{2.236} + 1 \right]^{-1}$$

b) Compute value at low-frequencies

There is one zero at zero. The bode plot will go to zero asymptotically with a slope of 20 dB/dec as $\omega \to 0$

Compute a point on the asymptote for some value of ω. We choose $\omega = 0.022$ because this is two orders of magnitude smaller than the smallest pole or zero.

$$20 \log_{10} G(j0.02236) = 20 \log_{10} 0.2 + (1) 20 \log_{10}(j0.02236) = -46.99 dB$$

c) Compute the phase at low frequencies

$$\begin{aligned}
\lim_{\omega \to 0} \angle G(j\omega) =& 0 && 0^\circ \text{ if } K_0 > 0 \text{ else } 180^\circ \\
& - 90 \cdot 0 && -90 \cdot \text{(Num integrators)} \\
& + 90 \cdot 1 && 90 \cdot \text{(Num zeros at zero)} \\
& - 180 \cdot 0 && -180 \cdot \text{(Num of RHP poles)} \\
& + 180 \cdot 0 && 180 \cdot \text{(Num of RHP zeros)} \\
=& 90 &&
\end{aligned}$$

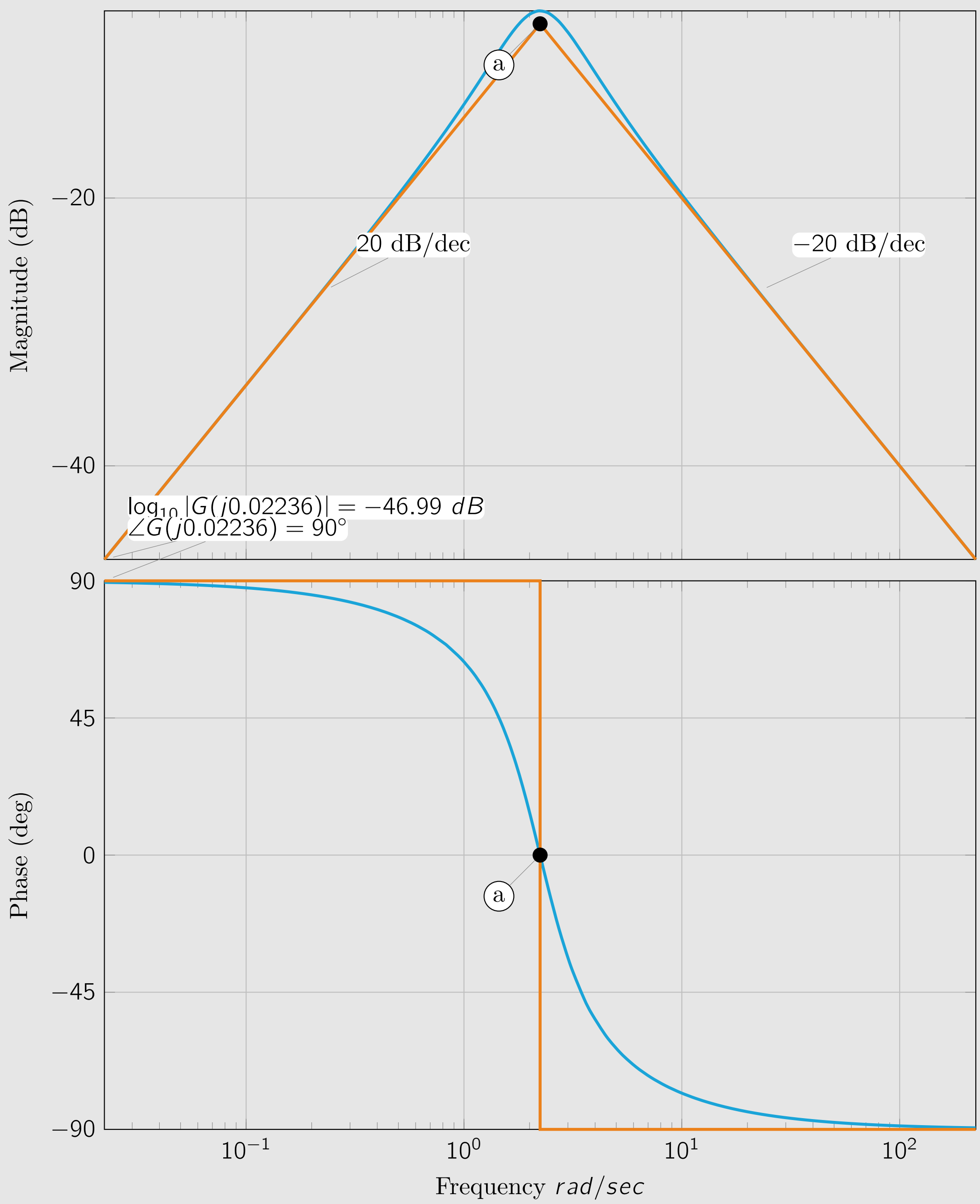
d) Compute impact of each pole and zero

Compute impact of each pole / zero on magnitude and phase from low to high frequency.

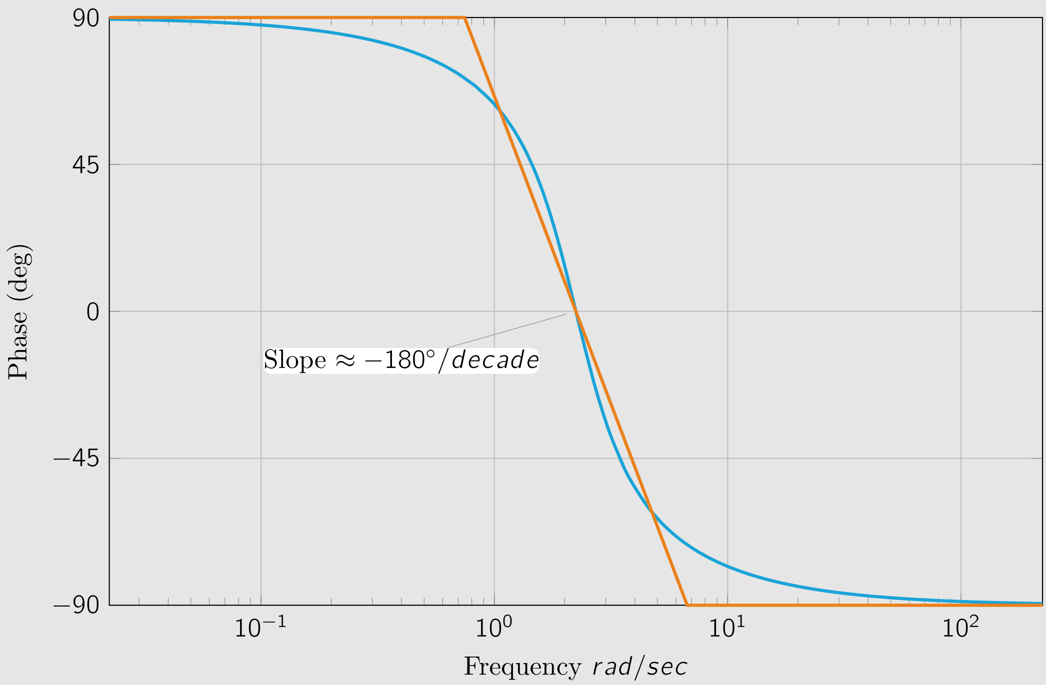
		Freq	Pole/Zero	Order	ΔdMag	ΔPhase
a	$\left[\left(\frac{jw}{2.236} \right) + 2 \cdot (0.4636) \frac{jw}{2.236} + 1 \right]^{-1}$	2.236	LHP pole	2^{nd}	-40	-180

e) Sketch straight-line approximation

Straight-line approximation (orange). True Bode plot (blue)



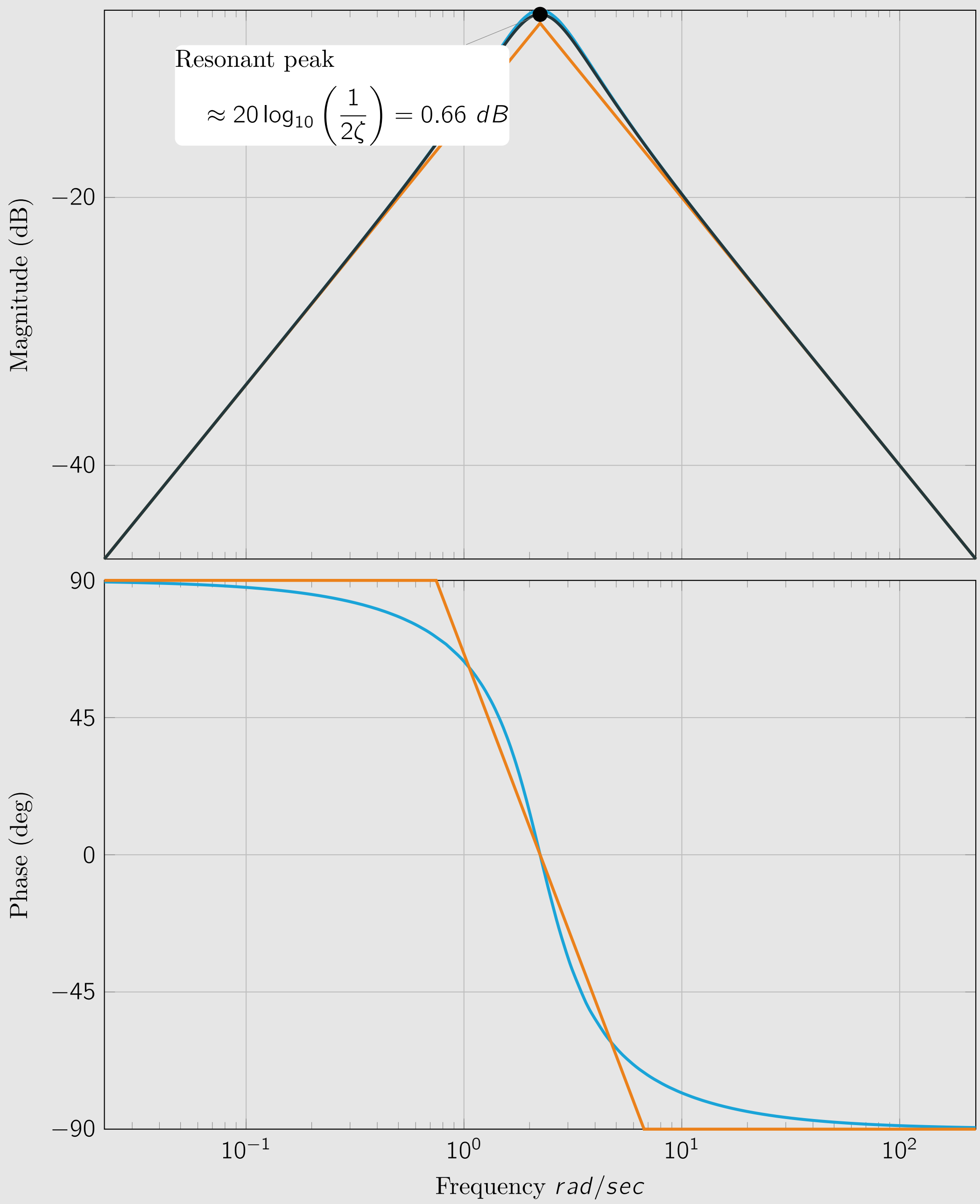
f) Improve accuracy of phase plot if required
Straight-line approximation (orange). True Bode plot (blue)



g) Add resonance peaks

For each second order pole and/or zero, compute the magnitude at the resonance peak

Straight-line approximation (orange). True Bode plot (blue). Straight-line approximation with resonant peaks (black).



Prob 5 | Sketch the bode plot for the system

$$G(s) = \frac{1}{(s-5)(s+2)}$$

Check your answer using the Matlab command bode

a) Put into Bode form

Order poles/zeros from lowest to largest frequency:

$$G(jw) = -0.1 \cdot \left(\frac{jw}{2} + 1\right)^{-1} \cdot \left(\frac{jw}{5} - 1\right)^{-1}$$

b) Compute value at low-frequencies

There are no integrators / zeros at zero:

$$G(j0) = K_0 = -0.1$$
$$20\log_{10}|G(j0)| = -20dB$$

c) Compute the phase at low frequencies

$$\begin{aligned} \lim_{\omega\to 0} \angle G(j\omega) =& 180 & 0^\circ \text{ if } K_0 > 0 \text{ else } 180^\circ \\ & -90\cdot 0 & -90\cdot(\text{Num integrators}) \\ & +90\cdot 0 & 90\cdot(\text{Num zeros at zero}) \\ & -180\cdot 1 & -180\cdot(\text{Num of RHP poles}) \\ & +180\cdot 0 & 180\cdot(\text{Num of RHP zeros}) \\ =& -180 & \end{aligned}$$

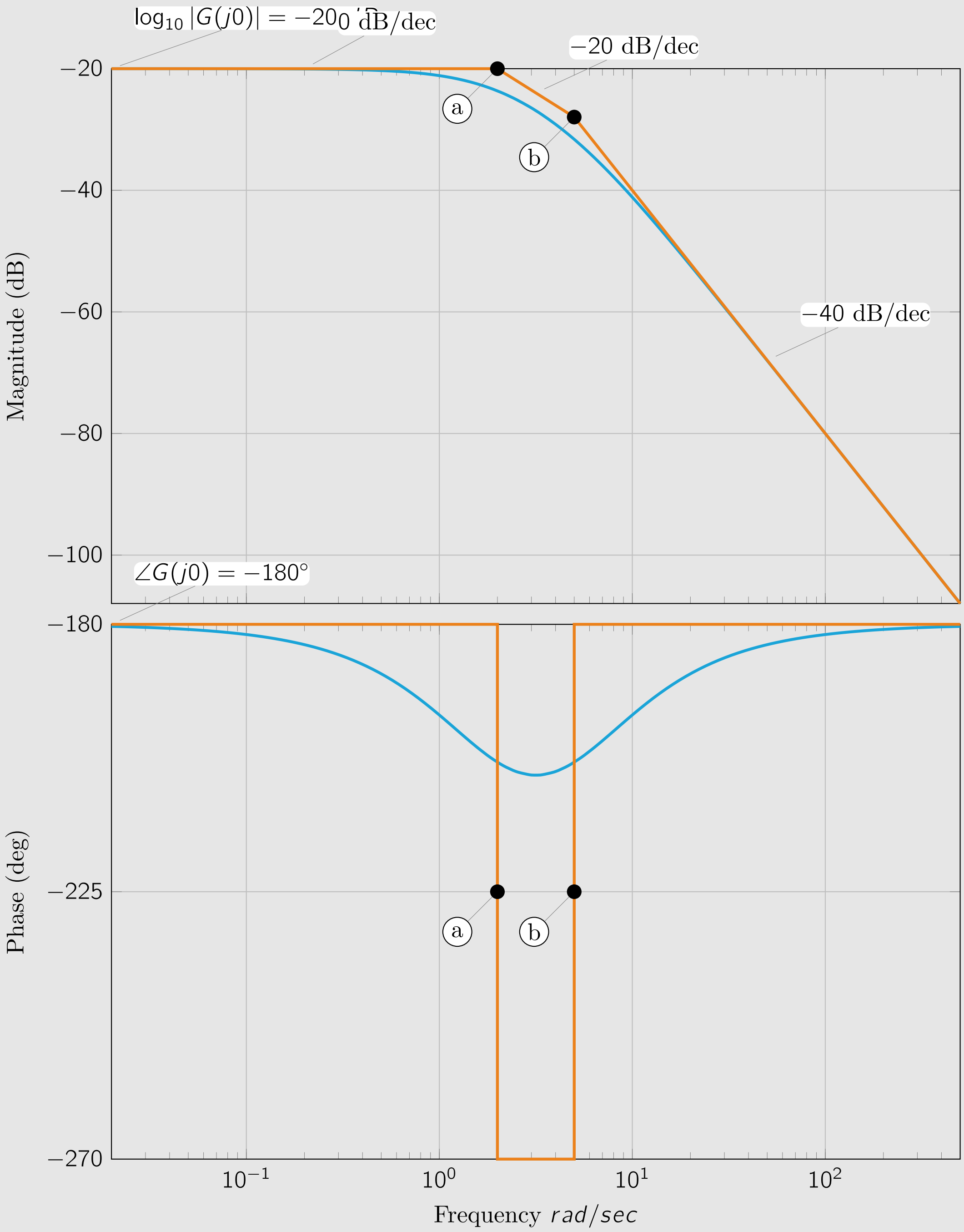
d) Compute impact of each pole and zero

Compute impact of each pole / zero on magnitude and phase from low to high frequency.

		Freq	Pole/Zero	Order	ΔdMag	ΔPhase
a	$\left(\frac{jw}{2}+1\right)^{-1}$	2	LHP pole	1^{st}	-20	-90
b	$\left(\frac{jw}{5}-1\right)^{-1}$	5	RHP pole	1^{st}	-20	+90

e) Sketch straight-line approximation

Straight-line approximation (orange). True Bode plot (blue)



f) Improve accuracy of phase plot if required

Straight-line approximation (orange). True Bode plot (blue)